Assessment Problem 1.4

1.4 The expression for the charge entering the upper terminal of Fig. 1.5 is

$$q = \frac{1}{\alpha^2} - \left(\frac{t}{\alpha} + \frac{1}{\alpha^2}\right)e^{-\alpha t}\ \mathrm{C}.$$

Find the maximum value of the current entering the terminal if $\alpha = 0.03679\ \mathrm{s}^{-1}$.

$$i = \frac{dq}{dt} = -\left[\frac{1}{\alpha}e^{-\alpha t} + \left(\frac{t}{\alpha} + \frac{1}{\alpha^2}\right)e^{-\alpha t} \times -\alpha\right]$$

$$i(t) = -\frac{e^{-\alpha t}}{\alpha} + \left(t + \frac{1}{\alpha}\right)e^{-\alpha t}$$

To find max i: $\frac{di}{dt} = 0$

$$\frac{di}{dt} = e^{-\alpha t} + \left[e^{-\alpha t} + \left(t + \frac{1}{\alpha}\right)e^{-\alpha t} \times -\alpha\right]$$

$$= e^{-\alpha t} + e^{-\alpha t} - (\alpha t + 1)e^{-\alpha t}$$

$$\therefore e^{-\alpha t}\left[2 - \alpha t - 1\right] = 0$$

$$\therefore 1 - \alpha t = 0$$

$$t = \frac{1}{\alpha}$$

$$i\left(\frac{1}{\alpha}\right) = -\frac{e^{-\alpha\left(\frac{1}{\alpha}\right)}}{\alpha} + \left(\frac{1}{\alpha} + \frac{1}{\alpha}\right)e^{-\alpha\left(\frac{1}{\alpha}\right)}$$

$$= -\frac{e^{-1}}{\alpha} + \frac{2}{\alpha} e^{-1}$$

$$= \frac{e^{-1}}{\alpha}$$

$$= \frac{1}{\alpha e}$$

$$= \frac{1}{0{,}03679 \times 2{,}71828}$$

$$= 10 \text{ A}$$